Numerical Methods	Code & No:	Math 254
	Credits:	3 (3,0,1)
	Pre-requisite:	MATH 205
	Co-requisite:	None
	Level:	8

Course Description:

This course will emphasize the development of numerical algorithms to provide solutions to common problems formulated in science and engineering. The primary objective of the course is to develop the basic understanding of the construction of numerical algorithms, and perhaps more importantly, the applicability and limits of their appropriate use.

Course Aims:

The emphasis of the course will be the thorough study of numerical algorithms to understand:

1. The guaranteed accuracy that various methods provide.
2. The efficiency and scalability for large scale systems.
3. Issues of stability.

Topics include the standard algorithms for numerical computation: root finding for nonlinear equations, interpolation and approximation of functions by simpler computational building blocks (for example - polynomials and splines), numerical differentiation and divided differences, numerical quadrature and integration, Linear Systems: Solution by Iteration, Gauss–Seidel Iteration, Convergence and Matrix Norms, Test of Convergence of the Gauss–Seidel Iteration, Jacobi Iteration, Linear Systems: Ill-Conditioning, Norms, Matrix Eigenvalue Problems, Bounds for Eigenvalues from Collatz's Theorem, numerical solutions of ordinary differential equations and boundary value problems, numerical optimization and regularization algorithms.

Student Outcomes (SOs):

☒(a) An ability to apply knowledge of computing and mathematics appropriate to the program's student outcomes and to the discipline

☒(b) An ability to analyze a problem, and identify and define the computing requirements appropriate to its solution

☒(c) An ability to design, implement, and evaluate a computer-based system, process, component, or program to meet desired needs

☐(d) An ability to function effectively on teams to accomplish a common goal

☐(e) An understanding of professional, ethical, legal, security and social issues and responsibilities

☐(f) An ability to communicate effectively with a range of audiences

☐(g) An ability to analyze the local and global impact of computing on individuals, organizations, and society

☐(h) Recognition of the need for and an ability to engage in continuing professional development

☒(i) An ability to use current techniques, skills, and tools necessary for computing practice.

☐(j) An ability to apply mathematical foundations, algorithmic principles, and computer science theory in the modeling and design of computer-based systems in a way that demonstrates comprehension of the tradeoffs involved in design choices. [CS]

☐(k) An ability to apply design and development principles in the construction of software systems of varying complexity. [CS]

☐(j) An ability to use and apply current technical concepts and practices in the core information technologies of human computer interaction, information management, programming, networking, and web systems and technologies. [IT]

☐(k) An ability to identify and analyze user needs and take them into account in the selection, creation, evaluation, and administration of computer-based systems. [IT]

☐(l) An ability to effectively integrate IT-based solutions into the user environment. [IT]

☐(m) An understanding of best practices and standards and their application. [IT]

☐(n) An ability to assist in the creation of an effective project plan. [IT]

Learning Outcomes:

Upon successful completion of the course, students should be able to:

1. Apply standard techniques to analyze key properties of numerical algorithms such as stability and convergence,
2. Understand and analyze common pitfalls in numerical computing such as ill-conditioning and Instability,
3. Perform data analysis efficiently and accurately using data fitting methods,
4. Derive and analyze numerical methods for ODEs and PDEs,
5. Perform optimization using well-established algorithms,
6. Implement a range of numerical algorithms efficiently in Mat lab.

SOs and CLOs Mapping:

CLO/SO	a	b	c	d	e	f	g	h	i	j	k	l	m	n
CLO1	√	√	√						√					

CLO2	√	√	√						√				
CLO3	√	√	√						√				
CLO4	√	√	√						√				
CLO5	√	√	√						√				
CLO6	√	√	√						√				

No.	Topics	Weeks	Teaching hours
1	Topics include the standard algorithms for numerical computation: Programming and Software	3	9
	Root finding for nonlinear equations: Simple Fixed-Point Iteration		
	The Bisection Method		
	The False-Position Method		
	Brent's Method		
	Multiple Roots		
	Systems of Nonlinear Equations: Newton-Raphson		
2	Interpolation: Newton's Divided-Difference Interpolating Polynomials	2	6
	Lagrange Interpolating Polynomials		
	Coefficients of an Interpolating Polynomial		
	Spline Interpolation		
3	Numerical differentiation: High-Accuracy Differentiation Formulas	2	6
	Richardson Extrapolation		
	Partial Derivatives		
	Derivatives of Unequally Spaced Data		
	Numerical Integration		
4	Linear Systems: Solution by Iteration, Gauss–Seidel Iteration, Convergence and Matrix Norms, Test of Convergence of the Gauss–Seidel Iteration, Jacobi Iteration,	1	3
5	Linear Systems: Ill-Conditioning, Norms, Matrix Eigenvalue Problems, Bounds for Eigenvalues from Collatz's Theorem	1	3
6	Numerical solutions: Ordinary Differential Equations	3	9
	Euler's Method		
	Improvements of Euler's Method		
	Runge-Kutta Methods		
	Solution of Systems of Equations		
	Adaptive Runge-Kutta Methods		
	Boundary Value Problems: The Shooting Method		

	Boundary Value Problems: Finite-Difference Methods		
	Eigen Value and Eigen Vector		
7	Numerical optimization: One-Dimensional Unconstrained Optimization	2	6
	GOLDEN-SECTION SEARCH, Newtons Method		
	Multidimensional Unconstrained Optimization: Random Search		
	GRADIENT METHODS		
	Steepest Ascent Method		
	Total	**14**	**42**

Textbook:

- Steven C. Chapra and Raymond P. Canale "Numerical Methods for Engineers" Mc Graw Hill, 7th Edition (2015).

Essential references:

- Leader, Jeffery J. Numerical Analysis and Scientific Computation. Addison Wesley. (2004).
- Erwin Kreyszig "Advanced Engineering Mathematics" John Wiley & Sons, Inc. 10th Edition (2011)
- James F. Epperson "An Introduction To Numerical Methods And Analysis" by John Wiley & Sons 2nd Edition, 2013